Advanced Engineering Mathematics
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Lecture No. # 25
Applications of Laplace Transform to Integral Equations and ODEs

Welcome back to series of lecture on Transform calculus and in the last lecture we have evaluated Laplace and inverse Laplace transform of various special function and today some of the function will be used to solve the differential and integral equations. So, what is, go for application and today we will continue with this by solving various kind of differential equations mainly the initial value problems, boundary value problems and also some integral equations and let us see.

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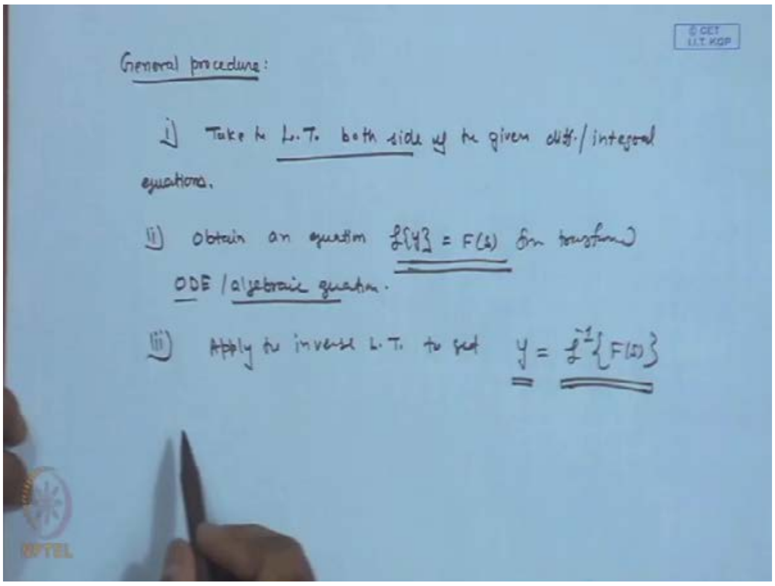

So, I start with the general procedure. So, the general procedure is that first we take the take the Laplace transform both side of the given differential or integral equation differential or integral equation. In the second step after taking this Laplace transform, we will get either algebraic equation or ordinary differential equations. In any case we will obtain an equation of the form Laplace of y as a function of s from this transformed differential equation, transformed ordinary differential equation or algebraic equation.

And third step, we will apply the inverse Laplace transform inverse Laplace transform to get the solution of the original problem. So, to get as y is Laplace inverse of F s So, y is our unknown differential equation. So, there are basically three steps in the; first we take Laplace transform both side of the given differential equation and then from the algebraic equation either we simply that algebraic equation or we solve the ordinary differential equation and we get the Laplace y is equal to function of this s. And then at last we apply the inverse transform and we get the solution of the original problem. So, we will discuss any with the help of various examples today in this lecture and let me just point out here that in the process we assume other solution is continuous and of exponential or because that is very important for the existence of Laplace transform. In fact, if our equation is linear and with constant coefficients then one can prove that solution is some assumption that the solution is continuous and exponential or. So, whatever example here we will consider in all of them we have the continuity in the solution and the solution is of exponential order. So, we can apply this Laplace transform without any problem.

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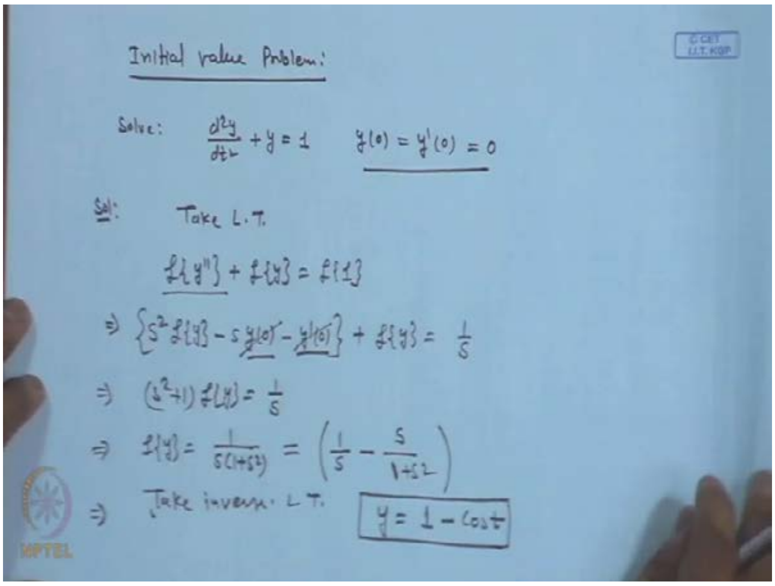

So, let us start with initial value problem initial value problem. So, in this problem we want to solve d2y over dt square plus y is equal to 1 and the given initial conditions are y at 0 and first derivative of y at 0, both are 0. So, very simple initial conditions. So, while solving as first step we take Laplace transform both side of equation. So, we have the Laplace transform of y double prime plus Laplace transform of y and the Laplace transform of one. By the derivative theorem and that is the point that we can use this initial value problems. So, that this derivative theorem will help. So, here if we apply the derivative theorem we have s square and Laplace of y minus s y 0 and minus y prime 0 and does see derivative theorem for this and then we have Laplace of y and we have Laplace of one that is one over s.

So, here this term is gone because is 0 and this is also 0. So, we have simply s square plus 1 and Laplace of y and this is equal to 1 over s and let me just point out here there is Laplace transform taken actually. This as well suited for solving the initial value problem actually because we have already use this initial value the given initial value use that for this is the best suited for initial value problem. So, here and now we get Laplace transform of y. So, everything as to right hand side. So, we have s and 1 plus s square and now we do partial fractions of this. So, we have simply in this case one over s then we have 1 plus s square. So, if we have here s simply and minus sign, we will get one over s and 1 plus s square and this is trick to get Laplace inverse basically by these partial fractions. So, we have one over s minus s over 1 minus s square, the Laplace of y. So, we take the inverse now take inverse Laplace transform and then we will get this y equal to. So, this Laplace inverse of one over s that is, one and we know the Laplace inverse of s over 1 minus s square is simply curve T. That is the solution which incorporate automatically the initial conditions because we are using at some point all these initial conditions.

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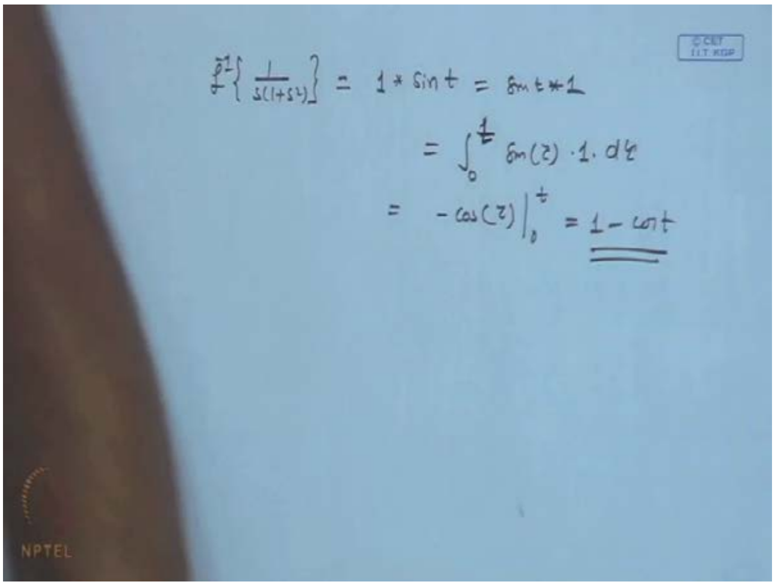

And solution just one more point that, we have valuated from the Laplace inverse of 1 over s 1 1 plus s square by the partial fraction method that the, I mean we did here partial fraction and then use this Laplace inverse to each fraction to get the Laplace inverse, but, what we can also do that using the convolution theorem we have discuss already in the last lecture. So, this is the convolution of the Laplace inverse of one over s and Laplace inverse of 1 over 1 plus s square. That is, so, this is convolution of 1 over s that is 1 and convolution with this sin t. That is Laplace inverse of 1 plus 1 over 1 plus s square. So, by this symmetric property, we have convolution one and this is nothing our 0 to t, the convolution integral and then we have sin tau and t minus tau. So, this is independent of t. So, we have d tau and sin t whole give as minus this cos tau and then 0 to t. So, we have simply 1 minus cos t same value what we obtain there.

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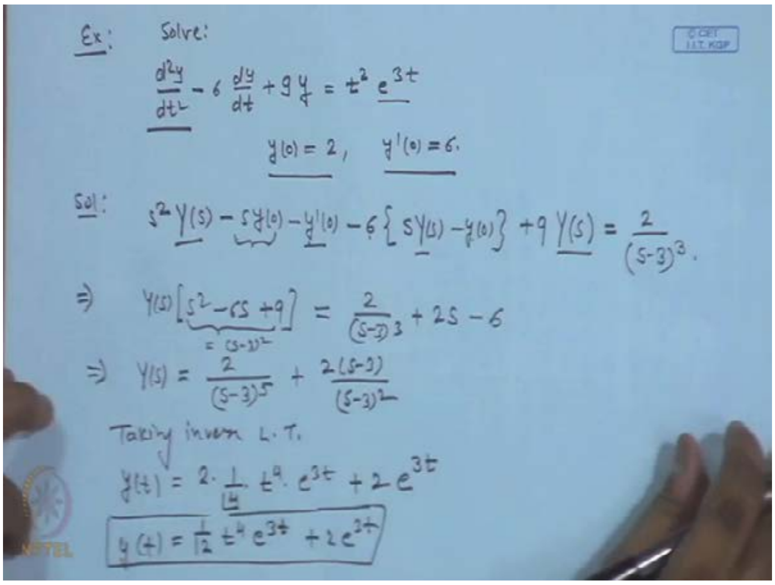

So, for the next example we solve d2y over dt square minus 6 dy over dt plus 9 y is equal to t square e 3 t and initial conditions are y 0 2 and y prime 0 is 6. So, slightly more difficult because say the initial values are, both initial values are given in the earlier example was 0. Our equation was much simplified up to taken Laplace transforms. So, here again we do the same steps, we take Laplace transform of this first term d2y over dt square and that is s square lap place transform of y. We can also denote by this big y s and minus s y 0 minus y prime 0 and minus 6. And here we can again use the derivative theorem s y s minus y 0 and plus 9 and Laplace of y that is y s and is equal to here the Laplace of t square e 3 t. So, use the first shifting property. So, Laplace of t square is 2 over s cube and then we have e power minus 3 t. So, the Laplace will be 2 over s minus 3 cube using the first shifting property. So, now, we collect the terms of this y s. So, what we have? s square from here minus 6 s and then plus 9, the right and side we have 2 over s minus 3 cube then y 0 is 2. So, this 2 s will go to the right side to give us 2 s. Then here we have 6 and then y 0 is 2.

So, we have plus 12 and minus 6. So, we get 6. So, that side it will be minus 6. From here we get this y s that is 2 over s minus 3 here if look at this; we have actually s minus 3 whole squares. So, then we take to right hand side here we have s minus 3 power 5 and 2 s minus 3 over s minus 3 square. So, now, the last step we take the inverse Laplace transform to get the solution and we have y t then and this case two. And the Laplace inverse of one over s minus 3 power 5. So, we have to use again the shifting theorem and that will give us e power 3 t and this is for t 4 1 over s power 4 s power 5. So, we have one over factorial 4, that constant will be moderated here and we have t 4 and e 3 t for this s minus 3 for shift and similarly, here this is 1 over s minus 3 and that will be e power 3 t. Again the shift theorem alternately one over s minus three we know e power three t. So, our solution is then y t is. So, here we have 1 over 12 and t 4 e 3 t plus 2 e 3 t. So, you get the solution again this incorporates the initial conditions we have. So, this was rather simple differential equation and now we will and that is mean application of this Laplace transform when the right hand side is basically it is continuous function or other complicated function sitting there and there very difficult to solve.

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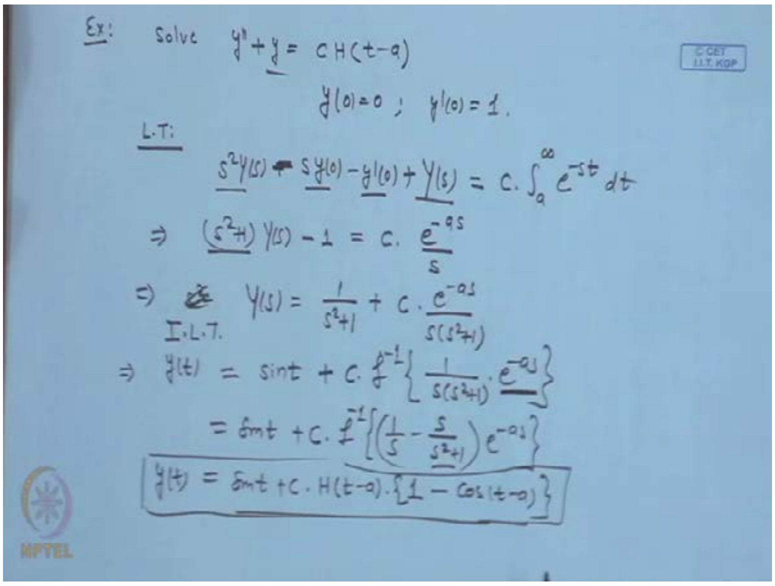

So, in the next example we will see then the right hand side solve y double prime plus y c and heavy side function t minus a. So, or unit step functions. So, here this constant when t is greater than a some positive and this is 0 and t is less than a. So, the initial conditions is given y prime 0 that is 1. So, again we take the Laplace transform. Both the side of equation s square y s plus sorry minus s y 0 and minus y prime 0. Here we have plus y s Laplace transform of y and then we have c, the Laplace transform of the second write from the basic definition. So, essentially this is defined for the first 0 a to infinity and it will be e power minus s t and d t. So, what we have here s square plus one and we have y s here y 0 is 0. So, this term is gone then and y prime 0 is 1. So, we have minus 1 here and are equal to c and now if integrate this e power minus s t over minus s and as t approaches to infinity for this positive s this will be 0. So, we have e power minus a s over s or we have s square. This we have take to right hand side we have y s is equal to 1 over s square plus 1 plus constant and e power minus a s over s and s square plus 1.

Now the last step we have take inverse Laplace transform. So, inverse Laplace transform and that will give us y t is equal to this we know that is sin t, the Laplace transform of one over s square plus 1 and we have constant term here. And the Laplace inverse of this one over s s square plus 1 and e minus a s. So, again here, sin t plus a constant and this second shifting theorem because e power minus a s is there then will be having shift in t. So, what we will get, but, first we need to know what is the Laplace inverse of this function? So, for that we need to first get the partial fraction of this 1 over s minus again s over s square plus 1 and we have e power minus a s and now we can do this, sin t plus c and this is the shift theorem we will be using here. So, heavy side function t minus a will come and Laplace inverse of this t minus a instead of t. So, we have here one there is no t 1 because Laplace inverse of 1 over s is 1 and minus this is cos t, the Laplace inverse s over s square plus 1 is cos t, but, we have this shift here. So, we will get cos t minus a. So, this is the solution of the given initial value problem.

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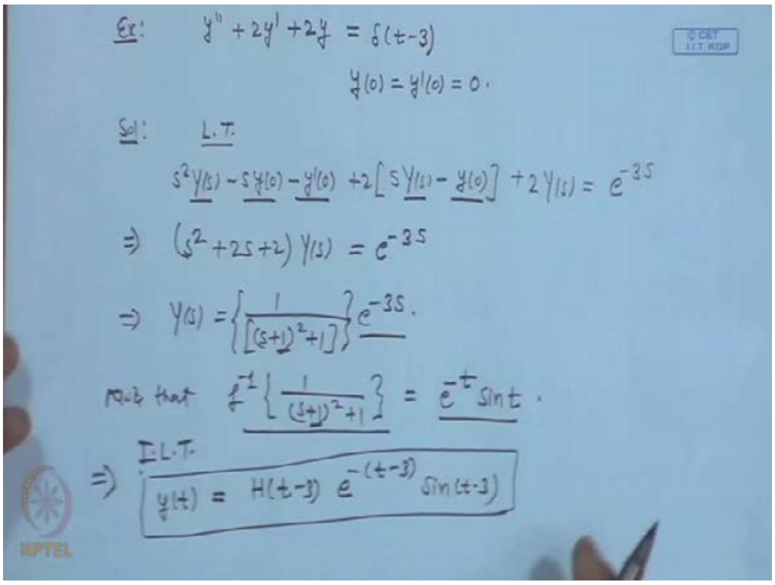

Now in the next example, we will take even more complicated right hand side of the differential equation and that is y double prime plus two y prime and 2y this equal to delta t minus 3. So, this delta function and we can easily solve this equation using the Laplace transform. So, the initial conditions we have y prime 0 y prime y 0 and both 0 just for simplicity and the solution we can see then. So, we apply Laplace transform both sides of the equation. So, we have s square Laplace transform of y minus s y 0 and we have minus y prime 0 plus this 2 again the derivative theorems. So, s y s minus y 0 and plus 2 we have y s and the Laplace transform of this delta function we have discussed in the last lecture that is simply e power minus 3 s. So, what we have now, we collect the coefficients of this y s. So, we have s square, we have 2 s and we have two from there and then y s. This y 0 this is 0 this is also 0 this is also 0. So, we have this all other term 0 and e minus 3 s. So, y s 1 over this we can also write s plus 1 whole square plus 1 and e minus 3 s.

Now, we this we can get using the calculation second shift theorem, but, first we need to get the Laplace transform of this term one over s plus 1 square plus 1 and this again the shifting theorem, the first shifting theorem because s shifted to s plus 1 because this is nothing as for the Laplace transform of sin t, but, we have to use shift theorem for this s plus 1. So, what we have. So, note that the Laplace inverse of this first s plus 1 whole square plus 1 using the first shifting theorem with e power minus t and the Laplace inverse of one over s square plus 1 and that is sin t. So, once we have this Laplace inverse e power minus t sin t we can get the Laplace inverse of this function with e power minus 3 s using in the second shifting theorem. So, this will give us taking the inverse Laplace transform now of the y s we will get y t and that will give us e power minus t sin t is there and we will get shift in t. So, with h t minus 3 because minus three is there because t minus 3 e power minus t minus 3 and sin t minus 3. With the second shifting theorem, we have this solution when the right hand side is t there delta function. So, these were examples of the initial value problems and now we will go for when the initial conditions are not given in that case also, we can get the general solution for example, we take this.

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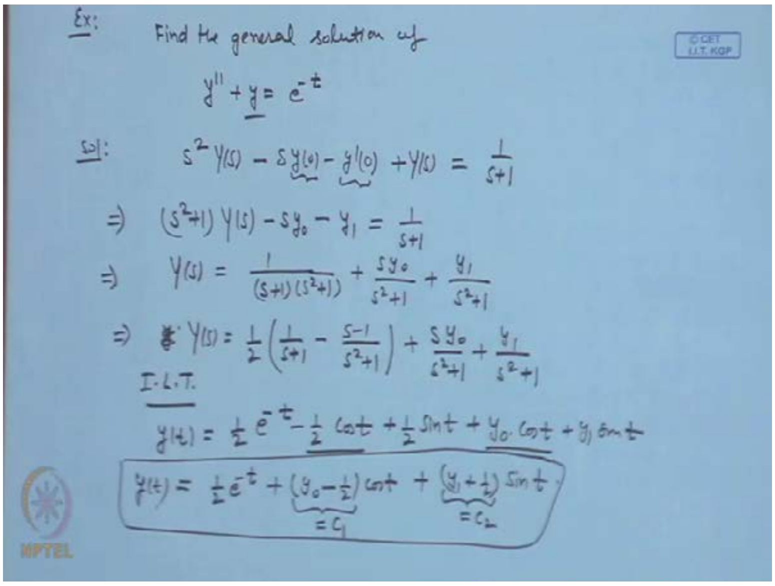

So, find the general solution the general solution of y double prime plus y e power minus t, no initial conditions are applied here. So, we have to get the general solution. So, now, the procedure is same. So, we have to take Laplace transform now. So, s square and y s minus s s y 0 minus y prime 0 and plus y s Laplace of y and Laplace of e power minus t is 1 over s plus 1. So, what we have now here s square plus 1 with this y s minus s. Let us call it y 0, a simple notation now for this we call it y 1 these are the constants and 1 over s plus one. So, instead of given some numerical value we will continue the calculation this y 0 and y 1. This is the only difference and we have then y s is equal to 1 over s plus 1 s square plus 1 plus s y 0 over s square plus 1 plus y 1 over s square plus 1 or y s. Again we do the partial fraction here. So, we have 1 over s plus 1 and s minus 1 over s square plus 1. So, what we have getting here? S square plus 1 minus s square and plus 1. So, we get 2 here. So, to compensate that we have half and plus s y 0 over s square plus 1 and plus y 1 over s square plus 1.

Now, we can take the Laplace Trans inverse Laplace transform. So, so inverse Laplace transform will give us now y t and this is 1 over 2 Laplace inverse of and this is one over two Laplace inverse of linearity property we can apply here. So, one over s plus one that will be e power minus t then we have again this half here minus half and s over s square plus 1 that is cos t. Then we have Laplace 1 over s square plus 1. So, this is sin t here again we have y 0 and with this cos t and we have y 1 and Laplace inverse 1 over s square plus 1 that is sin t. So, we have half e minus t and plus y 0 we can combine this two y 0 minus half cos t and plus this y one plus half and sin t and this y 0 y 1 are which constant here y 0 minus half we can put another constant and this we can put another constant. So, this is the general solution of the given differential equation. That is y t is half e power minus t some constant with cos t plus another constant and sin t. So, this was the example we have found the general solution of the differential equation.

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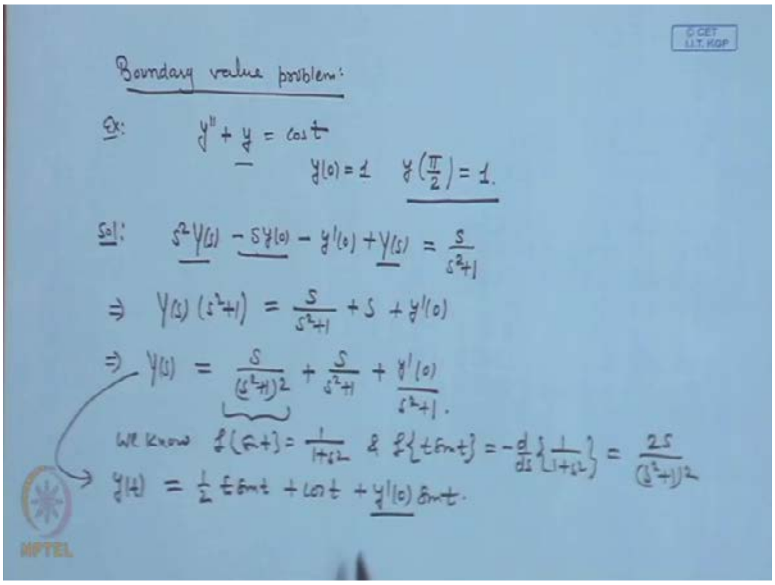

And now we continue for boundary value problem. Instead of initial value problems we have the boundary value problem boundary value problem. So, in this case we have y double prime plus y is equal to cos t and the conditions are given that y 0 is 1 and y at pi by 2 is 1. So, we do not have information on the first derivative at 0 instead of that we have the y at pi by 2 is 1. So, the procedure is same as the Laplace transform here. So, s square y s minus s y 0 and minus y prime 0 plus y s Laplace transform of this y and Laplace transform of cos t. That is s over s square plus 1.

So, we have y s now here and here and. So, with s square plus 1 and we have this y 0 this given that is 1. So, we can use that. So, this will take any way to the right hand side. So, we have s over s square plus 1 we have plus s from here and we have y prime 0 which is not known at this moment. So, here we get then the expression from y s this is s over s square plus 1 whole square plus s over s square plus 1 and plus y prime 0 over s square plus 1 and now, we can take the inverse. Before that we let me just point out that where our, we get s square plus 1 whole square type thing. So, because we know that Laplace transform of sin t is one over s square plus 1. So, we know that the Laplace transform of sin t that is 1 over 1 plus s square and also then Laplace transform of t sin t by theorem we have this d over ds of the Laplace transform of sin t. That is 1 over one plus s square and we get simply here get 2 s over s square plus 1 whole square. So, this is the term which we can see here. So if we take the Laplace inverse transform that y t whole be here and the factor will come. So, half t sin t plus cos t and here plus y prime 0 and then sin t. Now the question is how to get y prime 0, but, we have a condition here that y pi by 2 is 1. So, we can use that condition get this y prime 0.

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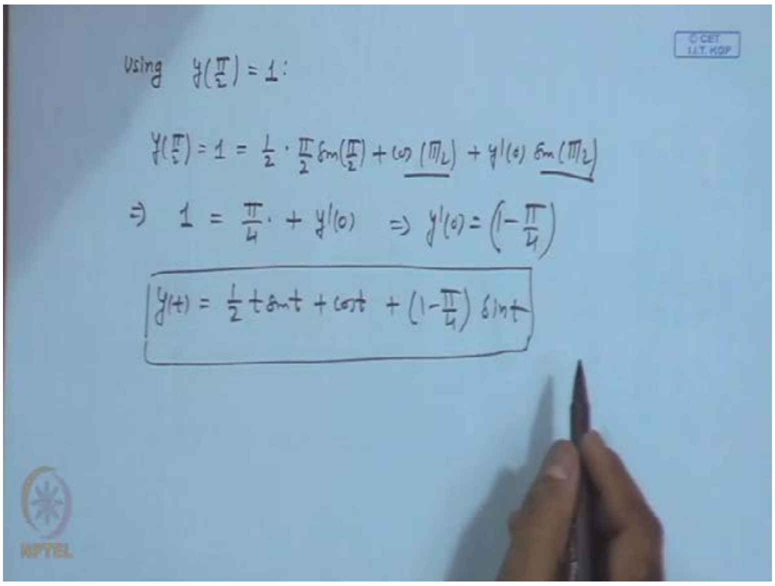

So, using the given boundary condition y pi by 2 is 1 we can have here y pi by 2 there is 1 and then half t. So, we have this solution again t sin t. So, we will get here pi by 2 and sin pi by 2 plus cos pi by 2 from this and then y prime 0 sin t. So, plus y prime 0 and sin t sin pi by 2. So, what we get here? One then we have pi by 4, this is 1 this is 0 and plus then y prime 0 this is also 1. So, here we get y prime 0 is 1 minus pi by 4 that we can again substitute to get that y t is half t sin t plus cos t plus 1 minus pi by 4 sin t. So, in this way we can also obtain the solution of the boundary value problem. Now, the next category we will move and here up to know we have the linear equation with constant coefficients. So, now, we will consider the differential equations when the constant of derivative terms are not constant, but, they are the polynomials.

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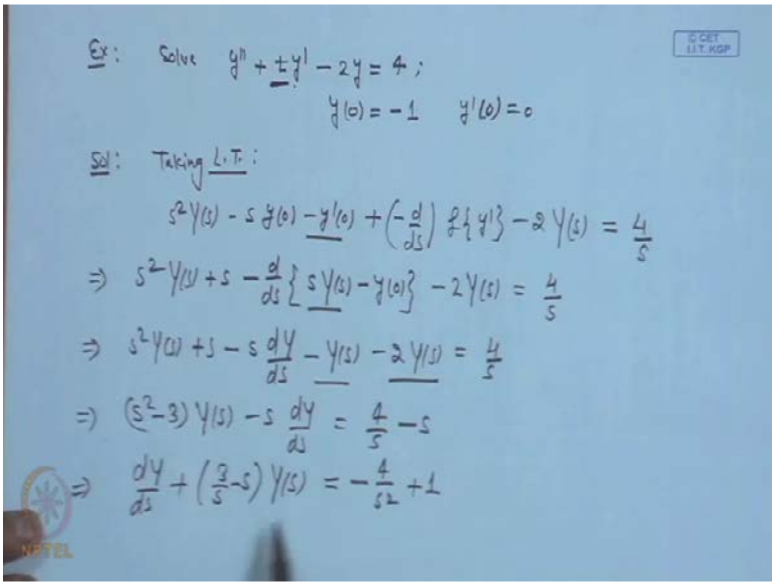

So, for example, solve this y double prime plus t y prime minus 2 y is equal to 4 and the initial conditions are given that y 0 is minus 1 and y prime 0 is 0. So, here this term, the coefficients are not constant any more, but, they are polynomial in t. The procedure is same, but, now this time we will get the ordinary differential equation rather than the simple algebraic equation. So, taking Laplace transform both sides what we get here? S square y s Laplace transform of y minus s y 0 and minus y prime 0 plus the Laplace of t y prime that is again the theorem we use when we multiplication by t. So, d over ds and the Laplace transform of y prime minus 2 Laplace transform of y and 4 Laplace transform of 1. So, 4 over s. So, it we have now?

First let us do this, y s minus s and y 0 is minus 1. So, we will get plus s this is 0. So, we have minus d over ds of d over d s of Laplace of y prime. So, that is s y s minus y 0 by the derivative theorem 2 y s 4 over s. So, s square y s plus s minus we differentiate here as product. So, d over ds we will get s d y over d s and will get minus y s here it is a constant term. So, d over d s will make it 0 and 2 y s we have 4 over s. So, we take common this y s here also 2 y s. So, we have s square minus three with y s and then we have minus s d y over d s we have right hand side four over s and we have this take to the minus s to the right side. So, what we have? The equation d y over ds ordinary differential equation plus 3 minus s because 1 minus sign will accommodate this 3 over s and minus s we have y s and is equal to dividing by this s. So, we have four over and multiplying this minus sign. So, 4 over s square and plus 1 plus 1. Now, this is the ordinary differential equation linear equation.

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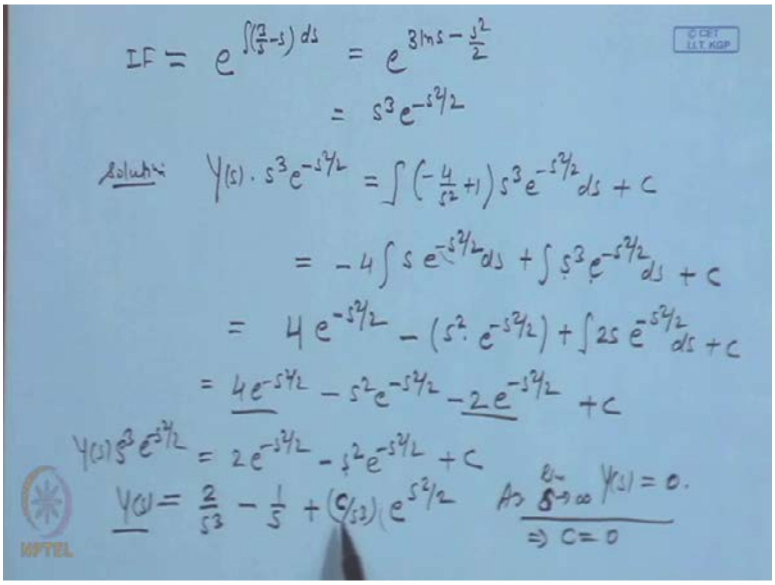

So, we will solve this. So, we get integrating factor integrating factor that will be e over e power 3 over s minus s d s and this is just e power minus s. So, 3 over s log s e 3 l n s minus s square by 2. So, this is here we can again simplify this will be just s cube and e minus s square by 2. So, the solution of that ordinary differential equation in y s will be s y s integrating factor e minus s square by 2, the integral and right hand side that we have here minus 4 s square plus 1. So, minus 4 over s square plus 1, this integrating factor e square e minus s square by 2 d s and a constant of integration. So, integrate this. So, we have minus 4, this you multiply, we have s e minus s square by 2 the first term then we have plus s cube e minus s square by 2 d s and a constant. So, this we can integrate because this differentiation of this is sitting here with minus sign we will adjust at this place. So, this is nothing else but four and e minus s square by 2 minus. Now, here we have to integrate by parts. So, this one will adjust s e power minus s square by 2 to get this integral. So, we have minus this s square the integration of s e minus s square by 2 will be e minus s square by 2 with minus sign that is here and then minus and then minus will come from that place. So, we have plus and we have 2 s the differentiation of this s square and again the integral of s e power minus s square by 2 will give minus and that is adjusted here. So, here that we have s square by 2 and d s plus c. So, 4 e minus s square by 2 minus s square e minus s square by 2 and we have this state now again.

So, we have minus 2 and this minus d 2, the integral will come with minus and then e minus s square by 2 and plus c. Now, this this will give us 2 e minus s square by 2 and we have minus s square e minus s square by two plus a constant. So, this is with y s and s 3 e minus square by 2. So, we get from here y s y s is equal to 2 over s cube 2 over s cube minus one over s and plus this c over s cube and e s square by 2. Now, again we see this one constant here. So, what will happen to this constant? We have already use our initial conditions. Here there is a point that this y s that is the Laplace transform of this y and this as a property as this is the Laplace transform of y s and exponential of function s as s approaches to infinity this must go to 0. As limit s approaches to infinity this s goes to infinity this y as must go to 0 and this is only possible if we see here e power s square by 2 sitting. If s approaches to infinity this term will blow up and then we cannot get the 0 as. So, this is possible only 1 c is 0. So, due to this condition we have that c must be 0. So, c must be 0.

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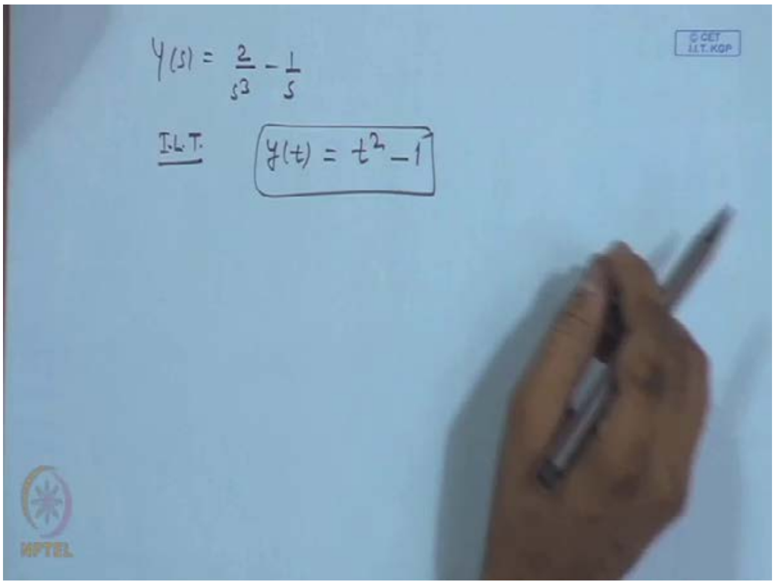

So, this term as gone then. Then we have y s this 2 over s cube minus s. Now, we take the inverse Laplace transform to get y t and this is Laplace transform 2 over s 3. There are simply t square and we have minus 1 this is the answer.

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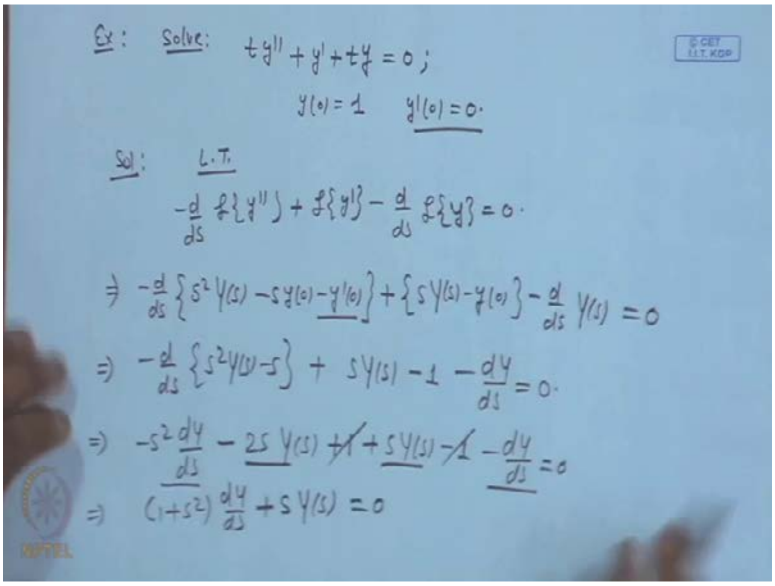

So, we take another example of a similar kind. So, we have to solve t y double prime plus y prime plus t y here also variable coefficients and this y 0 is 1 and y prime 0 is 0. Solution again; we take the Laplace transform. So, Laplace transform of this because of this t sitting. So, we have minus d over d s by the property of Laplace transform and the

Laplace over y double prime plus the Laplace of y double y prime minus again d over d s and the Laplace of Laplace of y, this is 0. So, minus d over d s and we have here s square Laplace of y minus s y 0 minus y prime 0 plus s y s minus y 0 and we have minus d over d s and we denote it by y s and is equal to 0. What we get here y prime 0 is 0. So, this term will vanish. So, we have minus d over d s square y s minus y 0 is 1 plus s y s minus y 0 is 1 plus s y s minus y 0 is 1 minus d y over d s is equal to 0.

So, we simplify this minus s square d y over d s minus 2 s y s and here we will get d over d s of s 1. So, minus, minus plus. So, 1 plus 1 plus s y s minus 1 minus dy over ds is equal to 0. So, we collect this terms of dy over ds. We have 1 plus s square that is multiply with minus sign sod y over d s. Then here, we have minus s y s. So, s y s multiplied by minus 1 and this is any way they cancel each other and then we have is equal to 0.

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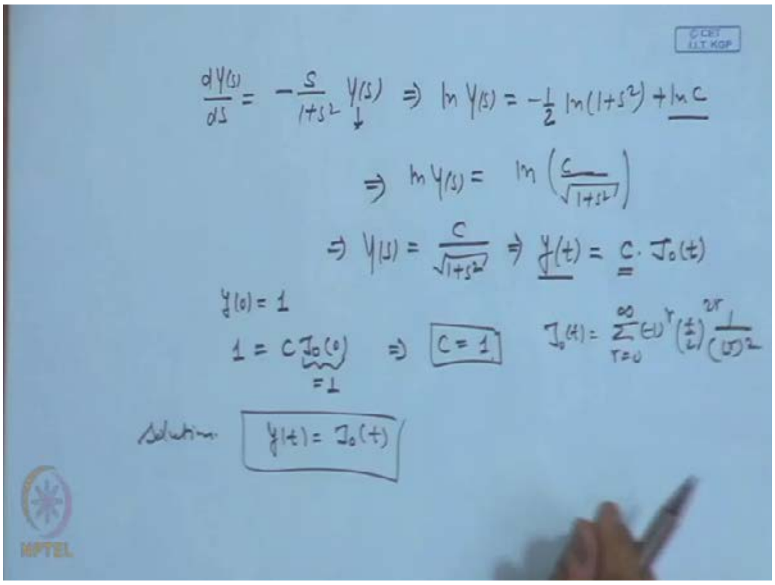

So, what we get the differential equation is d y s over d s is equal to minus s over 1 plus s square y s and this we can easily solve take this to the right hand side to the left hand side in the denominator. So, if we integrate that, we get l n y s is equal to here we will get this the differentiation is sitting there. So, with minus half and this l n 1 plus s square and constant l n c in this form because we will get rather simple form here. Then, l n c and divided by this square root of 1 plus s square. So, c over 1 plus s square root and here we have l n y s. So, in this way we get y s. This is c over one plus s square and if we take the inverse Laplace transform. So, we will get y t c and in the last lecture we have see in the Laplace inverse of 1 over 1 plus s square. That was best of function such j 0 t. So, what we have now? Now, we have the initial conditions are because this c appears now. So, what we can use at least that y 0 is 1 that is given. So, here we have y 0. So, we have 1 c and j 0 0 and j 0 0 we know that is one and in this way c we get is equal to one because remember this j 0 t was t was the series minus 1 r n t by 2 2 r 1 over factorial r whole square. So, the first term will be one and then all other term t appears as this is j zero will be one. So, we have c one. So, then the solution of that given differential equation is y t is j 0 t. So, that was the differential equation with variable coefficients and now we come to the integral equation.

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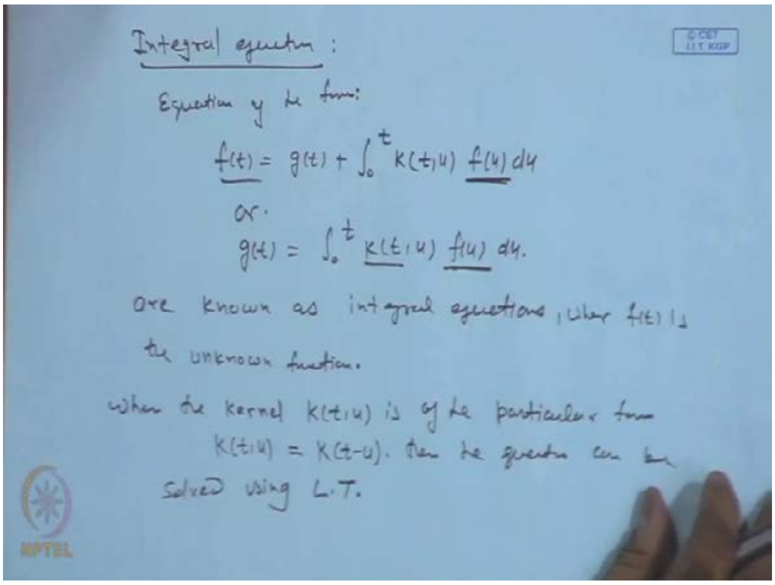

So, the integral equation here also we can apply the Laplace transform to get the solution. So, integral equation. So, equation of the form equation of the form f t is equal to g t plus 0 to t k t u f u d u or we can have other form k t is equal to this integral k t u and f u d u, these equations are known as integral equations integral equations where f t is the unknown function. So, here this f t is unknown, here f t is unknown. So, there is a special situation that when the kernel, that is called the kernel of integration, if this kernel k t u is of the particular form is of the particular form that this k t u is just function of t minus u then, the equation can be solved using the Laplace transform and we will see y because it will be simplify that integral which appears integral equations.

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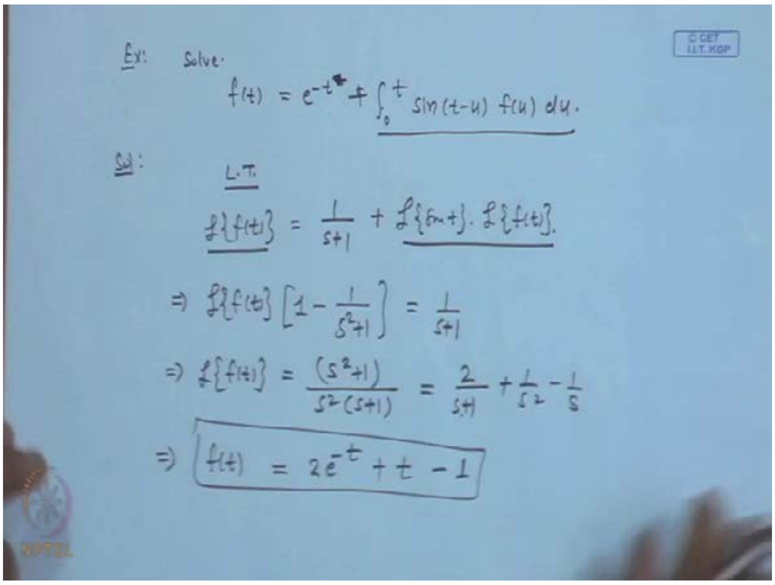

So, if we want to solve the integral equation f t e minus t square plus 0 to t sin t minus u f u d u; Solution: so, if we have apply the Laplace transform to this equation we will get the Laplace transform of this f t the Laplace transform of e minus t. This is as e minus t no t square. So, e minus t this Laplace transform is 1 over s plus 1 and plus. So, now, if we see this is the convolution integral. So, convolution of this sin t f t, so, if we take the Laplace of this convolution to will be just Laplace of the product. So, the Laplace of sin t and Laplace of f t by the Convolutional theorem and this is the point here. Now, it gets simplified. So, we take the Laplace transform f t then Laplace transform of f t and we have 1 minus this Laplace transform of sin t we know 1 over s square plus 1 and we have 1 over s plus 1 right hand side.

So, we get Laplace of f t is equal to s square plus 1 minus 1 s square this is nothing else right hand side we can take s square plus 1 over s square and s plus 1. Again, the partial fraction so, this step I skip. So, we have 2 over s plus 1 and plus 1 over s square and minus 1 over s. Now, we can take the Laplace inverse. So, we have 2 over s plus 1 that is 2 e power minus t 1 over s square t and then minus 1. So, this is the solution of that integral equation. Now, consider a different example when we will see the integral term as well as differential term. That is called integral differential equation.

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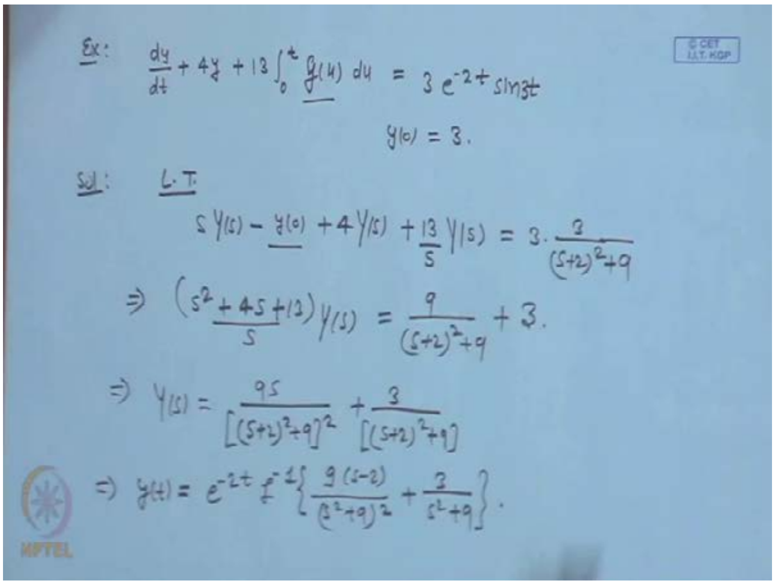

So, we take one example just for the make the concept clear. So, we have here the integral differential equation d y over d t plus 4 y plus 13 and 0 to t. We have y u and d u. This is 3 e minus 2 t and sin 3 t and y 0 because its differential term appears. So, we have to have the initial condition that is y 0 is given 3. So, now, we go along with same step. So, we have Laplace transform. So, here s the Laplace transform of y and minus y 0 plus this 4 Laplace transform of y and plus 13 and Laplace transform of this integral and that is just the, 1 over s and Laplace transform of this y. That was the property will discuss and Laplace transform then we have 3 and then we have the first shift property e power minus 2 t and sin 3 t for sin 3 the Laplace transform is 3 over s square plus 9. But, we have shifted here.

So, we have 3 over and s plus 2 instead of s because of e power minus 2 s square plus 9 now we collect the terms of this y s over s is here s square and then we have also 4 s and then we have 13 we have over s we have y s and this y 0 is 3. So, we can take to the right hand side which we have s plus 2 whole square plus 9 and plus plus we have this y 0 that is 3. So, we simplify this to get y s will be 9 s and this is again the same term because this is s plus 2 whole square. So, we have 4 and then 9. So, it is the same term. So, we get basically s plus 2 whole square plus 9 and this square and plus we have 3 over s plus 2 whole square plus nine. Now, we take the inverse Laplace transform and just remember again the shift property because s is s plus 2. So, let us apply shift property we get e minus 2 t and Laplace inverse of nine s minus 2 over s square plus 9 whole square plus 3 over s square plus 9. So, s is now s minus 1. So, we have s square s minus 2 and we have s square plus 9.

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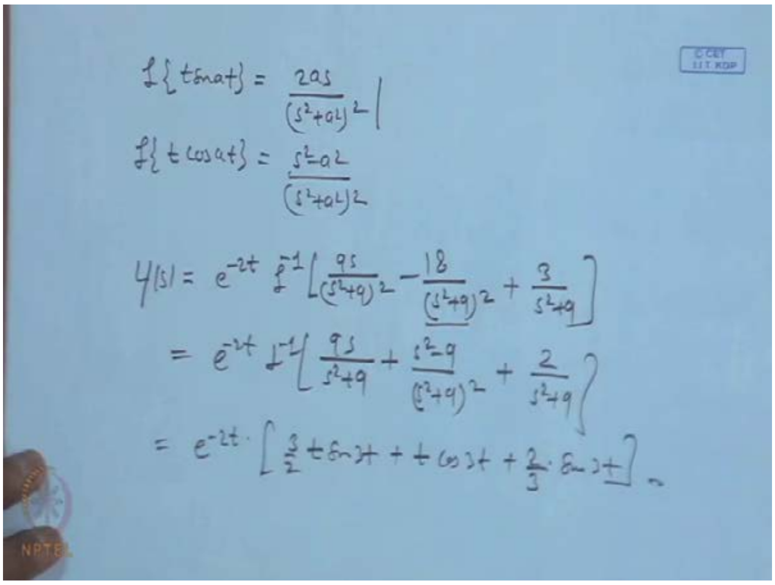

Now, to get this Laplace inverse we should note that the Laplace transform of t sin a t is 2 a s over s square plus a square just minus d over d s of sin a t Laplace of sin a t. Similarly, the Laplace transform of t cos a t is s square minus a square over s square plus a square whole square because it is whole square term is sitting there. So, we have that y s is e power minus 2 t and the Laplace inverse of 9 s over s square plus 9 whole square this is data we will get sin 3 t we have minus a t over s square plus 9 whole square and we have 3 over s square plus 9. So, we have get from this sin, we need to have here s square minus a square term. So, 2 t we have l inverse this 9 s we will keep it and this 3 we will take 1 over s square plus 9 we will combine with s and what you will get because 1 over s square plus 9 will be s square plus 9 by s square plus nine whole square. So, we will get simply s square minus 9 over s square plus nine whole square and plus this 2 over s square plus 9 because 1 over s square plus 9, we have combined with this term. So, we get e power minus 2 t and l inverse of this is easy now is s is given form 2 over s square plus a square by 2. So, this is simply 3 by 2 and t sin 3 t. Here we get t cos 3 t and here we get 2 by 3 and three sin 3 t. So, this is the solution. The last example we consider quickly and simultaneous differential equation. I will just give the idea how to solve simultaneous differential equation.

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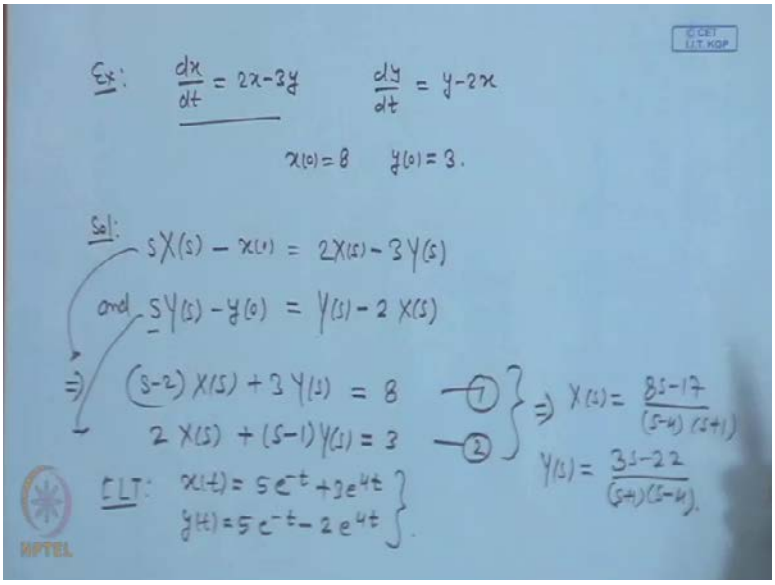

So, in this example we take very simple example; dx over d t 2 x minus 3 y and we have dy over d t that is y minus 2 x and the initial conditions must be given that x 0 is 8 and y 0 is given 3. So, what we do? The same steps; we take the Laplace transform of this two equations. We take Laplace transform this equation with denote by big x s s minus x 0 and 2 x s minus 3 y s and for the second equation we have, s y s minus y 0 we have y s minus 2 x s. So, now, from the first equation we have s minus 2 x s x 0 is given 8. So, will take to the right hand side and this 3 y s and then we have eight. Let us call it equation one and from this equation we will get this y s 2 x s 2 x s and we get s minus 1, this s minus 1 y s and is equal to 3. That is equation number two now we need to solve the 2 equations and I will skip that step is very easy here we multiply by s minus 1. So, example multiply by 3 then we can cancel this y as term we can eliminate this y s we can get this x s 8 s minus 12 over s minus 4 and s plus 1 and we get this y s we get this y s 3 s minus 22 over s plus 1 and s minus 4.

And now we can take the inverse Laplace transform of in both cases again the partial fractions we have to do and we will get straight away 5 e power minus t plus 3 e four t and this y t we will get 5 e minus t minus 2 e 4 t. So, it was just just an idea how to solve this, how to apply this Laplace transforms for system of equation and now I conclude this lecture. So, in this lecture we have seen that how we can apply this Laplace transform various kind of differential equations and we have mainly discuss the various initial value problems, boundary value problems, how to find general solutions and then integral equations; also for the integral differential equation and at last system of differential equations. So, in and the next lecture we will apply we continue the idea for solving the partial differential equations and that is all for today's. Thank you. Good bye.